# RAJAGIRI SCHOOL OF ENGINEERING \& TECHNOLOGY (AUTONOMOUS) <br> FIRST SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2022 <br> 101009/MA100 A 

## DISCRETE MATHEMATICS

Max. Marks: 100
Duration: 3 hours

## PART A

(Answer all questions, each question carries 3 marks)

1. Simplify the Boolean expression $a^{\prime} \cdot b^{\prime} \cdot c+a \cdot b^{\prime} \cdot c+a \cdot b^{\prime} \cdot c^{\prime}$, using Boolean algebra identities.
2. Prove that $a+\bar{a} b=a+b$
3. Define the relation R on the set of positive integers by $(x, y) \in R$ if the greatest common divisor of x and y is 1 . Determine whether R is reflexive, symmetric, antisymmetric, transitive, and/ or a partial order.
4. Find the cross product of sets $A=\{1,2,3\}, B=\{a, b\}$
5. Find the number of students in a class to be sure that four out of them are born on the same month.
6. Find the associated homogeneous solution for $a_{n}=3 a_{n-1}+2 n$.
7. Define Euler graph. Give an example.
8. Define Planar graph. State Euler formula.
9. Show that $(\neg p) \rightarrow(p \rightarrow q)$ is a tautology
10. Translate the sentences into propositional expressions:
"Neither the fox nor the lynx can catch the hare if the hare is alert and quick."

## PART B

(Answer one full question from each module, each question carries $\mathbf{1 4}$ marks)

## Module -I

11. a) Reduce the expression $a(a+c)=a a+a c$.
b) Discuss about Logic gates.
12. a) For the Truth table below, transfer the outputs to the Karnaugh, then write the Boolean expression for the result.

| $\mathbf{A}$ | $\mathbf{B}$ | Output |
| :--- | :--- | :--- |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |

b) Simplify the logic diagram below.


## Module -II

13 a) At Sunnydale High School there are 55 students in either algebra, biology, or chemistry class, 28 students in algebra class, 30 students in biology class, 24 students in chemistry class, 8 students in both algebra and biology, 16 students in both biology and chemistry, 5 students in both algebra and chemistry. How many students are in all three classes?
b) State any two properties of a group. Give an example of a subgroup.
14. a) Prove that (A,.) is a non abelian group where $A=R^{*} \times R \&(a, b) .(c, d)=(a c, b c+d)$
b) Define ring. Give examples.

## Module -III

15. a) $\operatorname{Let} \boldsymbol{S} \subset \boldsymbol{Z}^{+}$, where $|\boldsymbol{S}|=\mathbf{3 7}$. Show that $\boldsymbol{S}$ contains two elements that have the same remainder upon division by 36.
b) Find the recurrence relation for the sequence $a_{n}=2 n+9, n \geq 1$
16. a) Define an equivalence relation. Let m be a fixed positive integer. Two integers a and b are said to be congruent modulo $m$, written $a \equiv b(\bmod m)$ if $m$ divides $a-b$. Show that relation of 'congruence modulo $m$ ' is an equivalence relation.
b) Prove that $\mathbf{n}!>\mathbf{2}^{\mathbf{n}}$ for n a positive integer greater than or equal to 4 .

## Module -IV

17. Consider the following graph:

a. Find a Hamilton path. Can your path be extended to a Hamilton cycle?
b. Is the graph bipartite? If so, how many vertices are in each "part"?
18. a) Draw a graph with chromatic number 6 (i.e., which requires 6 colors to properly colour the vertices). Could your graph be planar? Explain.
b) Which of the following graphs contain an Euler path? Which contain an Euler circuit?
a) $\mathrm{K} 4 \quad$ b) K 5

## Module - V

19. a) Show the following equivalence, using truth tables: $P \Rightarrow Q \equiv Q \vee \neg P$
b) Determine whether the following arguments are valid or invalid:

## Premises:

a. If I read the newspaper in the kitchen, my glasses would be on the kitchen table.
b. I did not read the newspaper in the kitchen.

Conclusion : My glasses are not on the kitchen table.
20. a) Write the contrapositive, converse and inverse of the expressions: $P \rightarrow Q, \sim P \rightarrow Q, Q \rightarrow \sim P$
b) Show that the premises $E \rightarrow S, S \rightarrow H, A \rightarrow \sim H, \quad E \wedge A$ are inconsistent.

